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## 13. ABSTRACT (Maximum 200 words)

This report has presented a two-part method for estimating the directions of arrival of uncorrelated narrowband sources when there are arbitrary phase errors and angle-independent gain errors. The signal steering vectors are estimated in the first part of the method; in the second part, the arrival directions are estimated. It should be noted that the second part of the method can be tailored to incorporate additional information about the nature of the phase errors. For example, if the phase errors are known to be caused solely by element misplacement, the element locations can be estimated concurrently with the DOAs by trying to match the theoretical steering vectors to the estimated ones. Simulation results suggest that, for general perturbation, the method can resolve closely spaced sources under conditions for which a standard high-resolution DOA method such as MUSIC fails.

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# 1 Introduction

The ability of high-resolution direction of arrival (DOA) estimators such as MUSIC to distinguish between closely separated sources is severely degraded by perturbation errors (the term *perturbation error* refers to any discrepancy between the manifolds of actual and theoretical steering vectors for the array) [1]. Some possible causes of perturbation error are element misplacement, non-isotropy of the elements and/or the environment, inaccurate gain and phase calibration, and the assumption that the sources lie in the far-field. Various methods were proposed to deal with the problem of estimating DOA under perturbed conditions. Many of them made rather restrictive assumptions: [2] required at least two calibrating sources with known DOAs, while [3]-[4] required the sources to be disjoint in either frequency or time. The phase errors were assumed to be caused solely by element misplacement in [5]-[6], while in [7]-[9], the phase errors were assumed to be independent of angle (and therefore not caused by element misplacement). This report describes a new method for estimating the DOAs of non-disjoint, uncorrelated narrowband signals which makes no assumptions about the phase errors induced by the perturbation (like other methods which include phase errors in the perturbation model, it does make the standard assumption that the gain errors are angle-independent; the theoretical gains, however, may be non-isotropic if the signals are far-field). The first part of the method consists of an iterative algorithm which yields estimates of the signal steering vectors. In the second part of the method, these steering vectors are used to estimate the DOAs. This two-part structure and the generality of the perturbation model make the new method similar to the method described in [10]; the methods have not yet been compared.

## 2 Estimating the Steering Vectors

### Decomposing the signal covariance matrix

The algorithm starts by obtaining an estimate of the signal covariance matrix,  $\mathbf{R}_{ss}$ . If the noise covariance matrix is assumed to be a multiple of the identity matrix, an estimate can be found by replacing the small eigenvalues of the sample data covariance matrix with zeros after subtracting their average from the larger eigenvalues<sup>1</sup>.

Since the signals are uncorrelated, the total output signal power at an element equals the the total input signal power (which is the same for all elements) multiplied by the squared gain of the element. Consequently, the gains can be estimated by the square root of the diagonal components of  $\mathbf{R}_{ss}$ . Let  $g_k$  denote the estimated gain of the  $k$ th element, for  $k = 1, \dots, N$ . The signal covariance matrix is gain-calibrated by dividing the  $i, j$ th component of the matrix by  $g_i g_j$  for all  $i$  and  $j$ . After this

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<sup>1</sup>For an arbitrary noise covariance matrix,  $\mathbf{R}_{ss}$  can be estimated as described in the well-known paper on the MUSIC algorithm [11]

operation has been performed, all elements can be assumed to have unity gain.

Once the calibrated  $\mathbf{R}_{ss}$  has been obtained, its eigenvector decomposition is computed:

$$\mathbf{R}_{ss} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^{\dagger}, \quad (1)$$

where

$$\mathbf{U}^{\dagger}\mathbf{U} = \mathbf{I}, \quad (2)$$

and  $\mathbf{\Sigma}$  is a diagonal matrix.  $\mathbf{U}$  has dimension  $N$  by  $M$ , where  $M$  is the number of signals. The signal covariance matrix has another decomposition,

$$\mathbf{R}_{ss} = \mathbf{V}\mathbf{S}\mathbf{V}^{\dagger}, \quad (3)$$

where  $\mathbf{V}$  is a matrix of the signal steering vectors and  $\mathbf{S}$  is a diagonal matrix of the signal powers. From (1)–(3),

$$\mathbf{I} = \hat{\mathbf{U}}^{\dagger}\mathbf{V}\mathbf{S}^{1/2}\mathbf{S}^{1/2}\mathbf{V}^{\dagger}\hat{\mathbf{U}}, \quad (4)$$

where

$$\hat{\mathbf{U}} \triangleq \mathbf{U}\mathbf{\Sigma}^{-1/2}.$$

Since  $\hat{\mathbf{U}}^{\dagger}\mathbf{V}\mathbf{S}^{1/2}$  is a square matrix, (4) implies that

$$\mathbf{I} = \mathbf{S}^{1/2}\mathbf{V}^{\dagger}\hat{\mathbf{U}}\hat{\mathbf{U}}^{\dagger}\mathbf{V}\mathbf{S}^{1/2}.$$

Since  $\mathbf{S}$  is diagonal,

$$(\hat{\mathbf{U}}^{\dagger}\mathbf{v}_i)^{\dagger}(\hat{\mathbf{U}}^{\dagger}\mathbf{v}_j) = 0 \quad \text{for } 1 \leq i, j \leq M \text{ and } i \neq j, \quad (5)$$

where  $\mathbf{v}_i$  denotes the  $i$ th column of  $\mathbf{V}$ .

#### Updating the Estimated Steering Vectors

The algorithm attempts to find a set of vectors, represented by the columns of a matrix  $\hat{\mathbf{V}}$ , which satisfy (5).  $\hat{\mathbf{V}}$  can be initialized by setting its columns equal to the theoretical steering vectors that correspond to peaks in MUSIC (MUSIC is described in more detail in the following section). An arbitrary iteration of the algorithm begins by re-estimating the first steering vector while the other estimated steering vectors are kept fixed. The first transformed steering vector,  $\hat{\mathbf{U}}^{\dagger}\hat{\mathbf{v}}_1$ , should be orthogonal to all the other transformed steering vectors. Thus, it should lie in the null space of  $\hat{\mathbf{U}}^{\dagger}\hat{\mathbf{V}}_1$ , where  $\hat{\mathbf{V}}_1$  denotes the matrix formed by deleting the first column of  $\hat{\mathbf{V}}$ . This null space has a dimension of 1. For a vector  $\mathbf{z}$  in the null space,  $\hat{\mathbf{v}}_1$  should satisfy the under-determined set of equations

$$\hat{\mathbf{U}}^{\dagger}\hat{\mathbf{v}}_1 = \alpha\mathbf{z} \quad (6)$$

for some arbitrary scale factor  $\alpha$ . If  $\hat{\mathbf{v}}_1$  is a steering vector, it should lie in the column space of  $\mathbf{U}$  (see (1) and (3)). Thus,  $\hat{\mathbf{v}}_1$  should also satisfy

$$\hat{\mathbf{v}}_1 = \mathbf{U}\mathbf{q} \quad (7)$$

for some vector  $\mathbf{q}$ . Substituting (7) in (6) and making use of (2), one finds that

$$\hat{\mathbf{v}}_1 = \alpha \mathbf{U} \Sigma^{1/2} \mathbf{z}. \quad (8)$$

After  $\hat{\mathbf{v}}_1$  has been updated, other steering vectors are re-estimated in the same fashion; the iteration concludes with the update of the  $M$ th steering vector estimate. It is clear that at the end of the first iteration of the algorithm,  $\hat{\mathbf{V}}$  will have converged to a set of vectors which satisfy (5) and lie in the column space of  $\mathbf{U}$ . However, as it stands, the algorithm is incomplete. Additional structure must be imposed on  $\hat{\mathbf{V}}$ , corresponding to the fact that the absolute value of each component in  $\mathbf{V}$  is assumed to equal 1, the nominal gain of the calibrated elements. Thus, the last step in updating  $\hat{\mathbf{v}}_1$  consists of scaling it; its phase is left unchanged but its magnitude is replaced by 1. In the  $L_2$ -norm, the resulting vector is the one with the correct gains that is nearest to the unscaled  $\hat{\mathbf{v}}_1$ . After the scaling operation,  $\hat{\mathbf{v}}_1$  no longer satisfies (6) or (7). Consequently, the algorithm must be applied for an indefinite number of iterations. Two criteria given below can be used to evaluate the convergence of the algorithm; it will be seen that the algorithm does not necessarily converge monotonically with respect to either of these quantities. The average orthogonality of the transformed, scaled vectors is defined as

$$\sum_{i \neq j} \frac{\text{ang}(\hat{\mathbf{U}}^\dagger \hat{\mathbf{v}}_i, \hat{\mathbf{U}}^\dagger \hat{\mathbf{v}}_j)}{M^2 - M},$$

where the angle between complex-valued vectors  $\mathbf{x}$  and  $\mathbf{y}$  is defined as

$$\text{ang}(\mathbf{x}, \mathbf{y}) \triangleq \cos^{-1} \left( \frac{|\mathbf{x}^\dagger \mathbf{y}|}{|\mathbf{x}| |\mathbf{y}|} \right).$$

The average fractional amount that the scaled vectors lie in the column space of  $\mathbf{U}$  is defined as

$$\sum_i \frac{|\mathbf{P}_\mathbf{U} \hat{\mathbf{v}}_i|}{M |\hat{\mathbf{v}}_i|},$$

where a projection matrix  $\mathbf{P}_\mathbf{X}$  is defined by

$$\mathbf{P}_\mathbf{X} \triangleq \mathbf{X}(\mathbf{X}^\dagger \mathbf{X})^{-1} \mathbf{X}^\dagger.$$

The algorithm can be terminated when no steering vector changes significantly from one iteration to the next, or after a fixed number of iterations.

### 3 Estimating the Directions of Arrival

The theoretical steering vector  $\mathbf{v}_\theta$  is computed under the assumption of no gain or phase errors. MUSIC estimates the directions of arrival by finding the values of  $\theta$  which yield peaks in  $-10 \log_{10}(\mathbf{v}_\theta^\dagger(\mathbf{I} - \mathbf{P}_\mathbf{U})\mathbf{v}_\theta)$ , where  $\mathbf{I}$  is the identity matrix of dimension  $N$  by  $N$ . The corresponding theoretical steering vectors lie far from the noise subspace, or equivalently, close to the signal subspace.

Since the algorithm described in the first section provides estimates of the individual signal steering vectors, one can independently estimate the direction of arrival of each signal. The direction of arrival of the  $j$ th signal is estimated by finding the single value of  $\theta$  which maximizes  $-10 \log_{10}(\mathbf{v}_\theta^\dagger(\mathbf{I} - \mathbf{P}_{\hat{\mathbf{V}}_j})\mathbf{v}_\theta)$ .

### 4 Simulation Results

The experiment involved a circular array of six elements with  $\lambda/2$  spacing between adjacent elements. The noise was white and its variance was the same at all elements i.e. the noise power did not depend on the element gains. The element gains were nominally equal to unity but actually varied uniformly from 0.5 to 1.5. Both MUSIC and the new algorithm used the gain-calibrated  $\mathbf{R}_{xx}$ . The signal directions of arrival were  $\theta = -10^\circ, 0^\circ$ , and  $10^\circ$ ; each signal had SNR of 20 dB. Twenty trials were performed. In a trial, each element was randomly displaced; the  $x$  and  $y$  displacements were independent and uniform over  $\pm 0.3\lambda$ . The signal steering vectors were additionally perturbed by angle-independent phase errors of up to  $.03\lambda$ . Once the perturbed steering vectors were generated, the sample covariance matrix was randomly formed from 500 snapshots. The theoretical steering vectors were calculated at one-degree increments. The algorithm was allowed to run for 200 iterations. Only once in the 20 trials did MUSIC have three peaks in the general vicinity of the true directions-of-arrival. The mean values for the new method's direction-of-arrival estimates were  $-11.0^\circ, -0.5^\circ$ , and  $10.3^\circ$ . The standard deviations were  $1.8^\circ, 2.4^\circ$ , and  $1.5^\circ$ .

The following figures were generated using the data from the twentieth trial. Fig. 1 shows that the algorithm improved upon the initial MUSIC estimates of the signal steering vectors; each estimated steering vector was compared with the closest actual steering vector and the average angular separation was determined (the gain errors were not considered in this calculation). Fig. 2 shows that the transformed estimated steering vectors rapidly become orthogonal to each other after a few iterations, as desired. This is one way to estimate how well the algorithm is performing. Fig. 3 reveals a second way to estimate the algorithm performance—the estimated steering vectors should lie close to the signal subspace. Figs. 4 and 5 illustrate the direction-of-arrival spectra for MUSIC and the new method. In the case of the latter, the individual curves for the three signals have been superimposed on the same plot. Although MUSIC was not capable of resolving the closely spaced signals, the three directions-of-arrival are apparent in fig. 5.

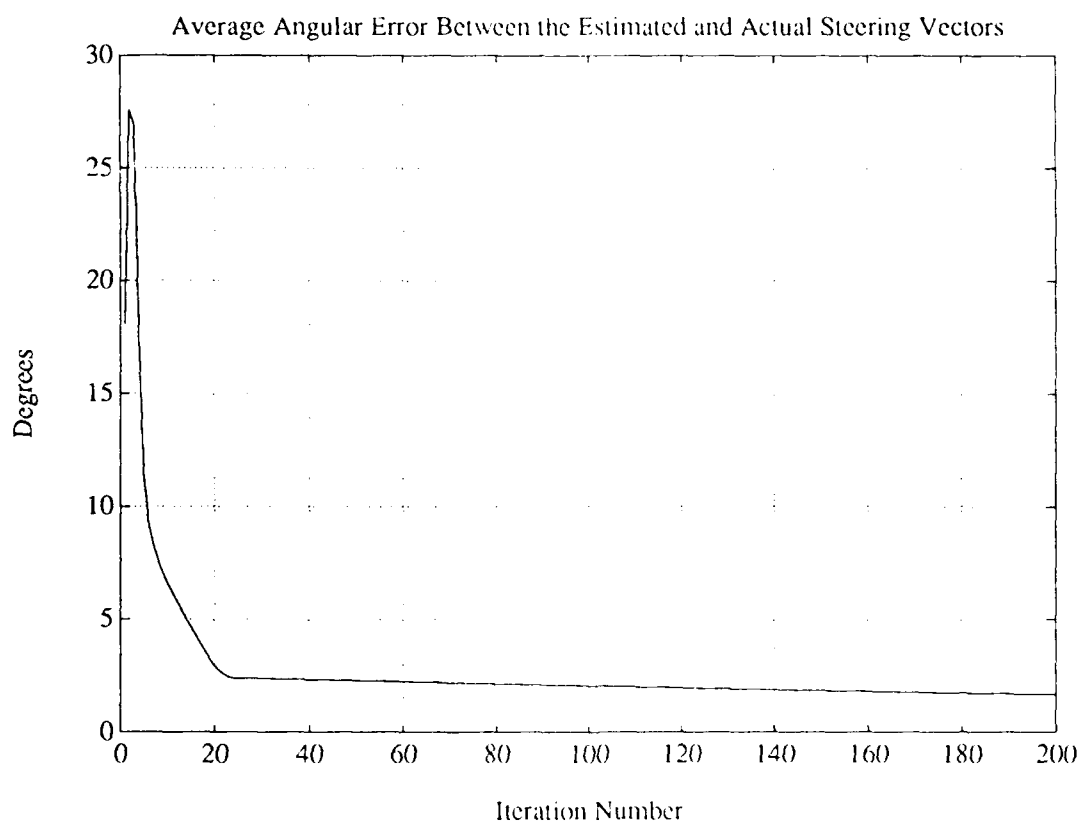


Figure 1: The Actual Error

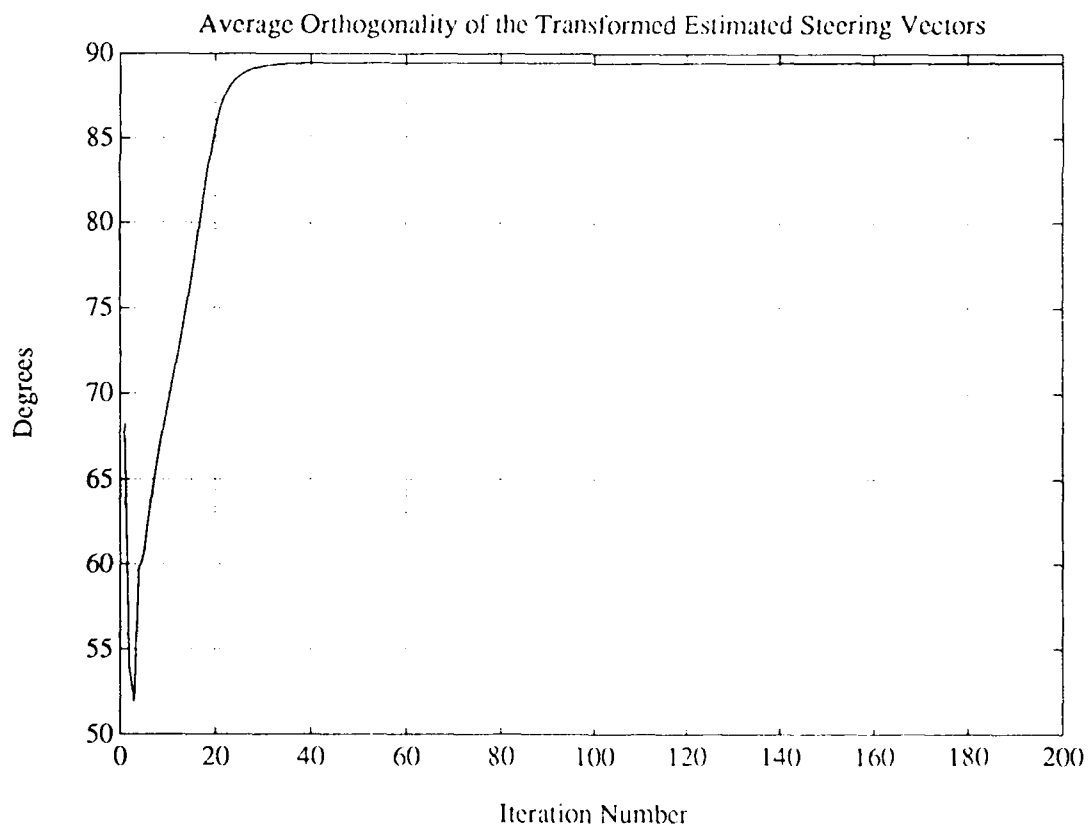


Figure 2: Error Estimate Based on Orthogonality

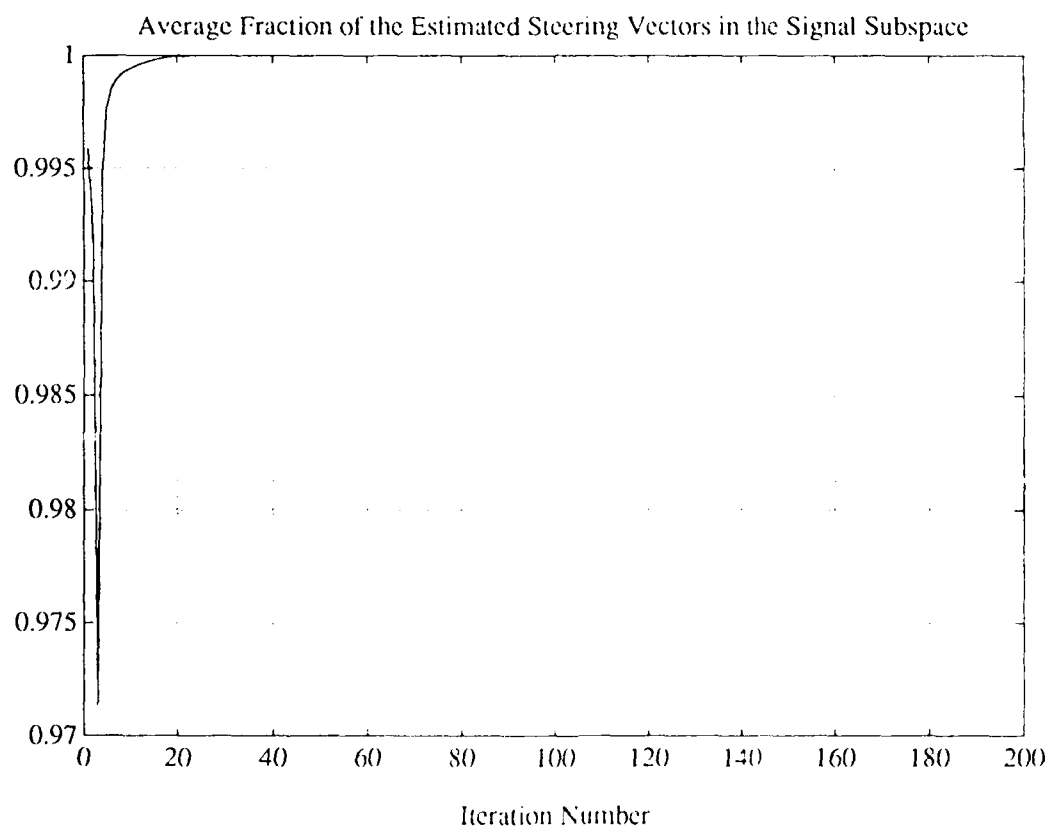


Figure 3: Error Estimate Based on Distance from Signal Subspace

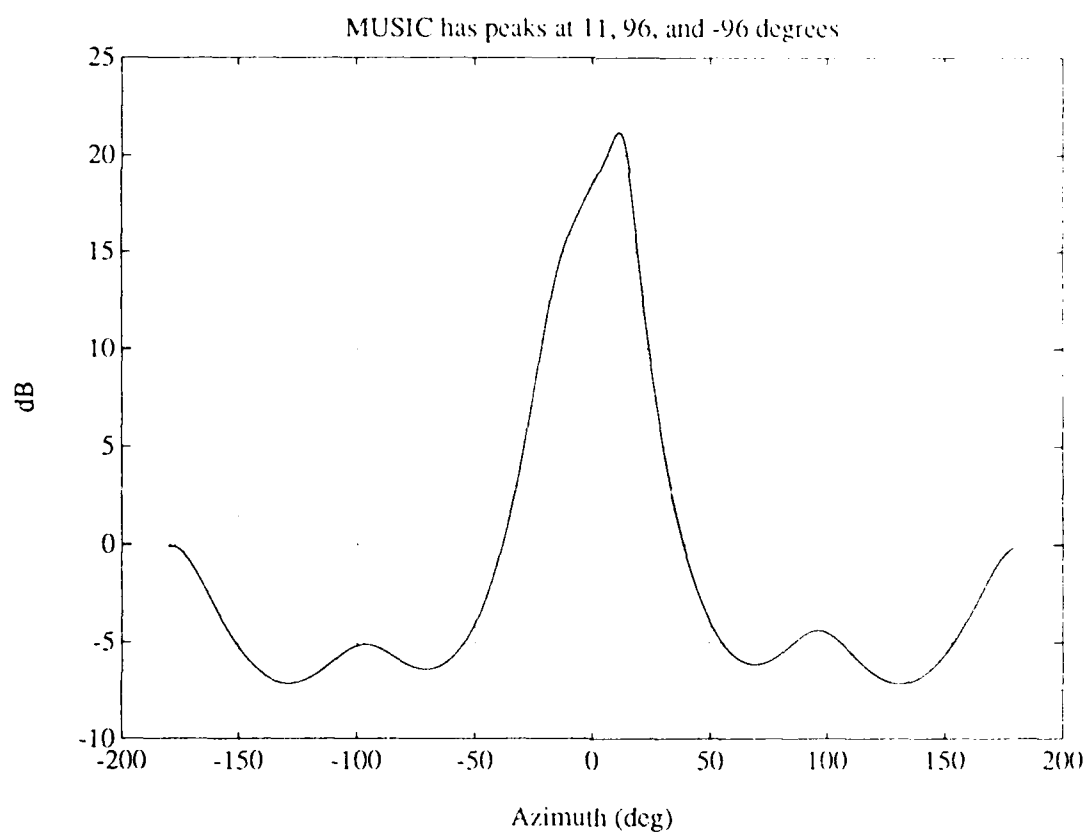


Figure 4: The MUSIC Spectrum

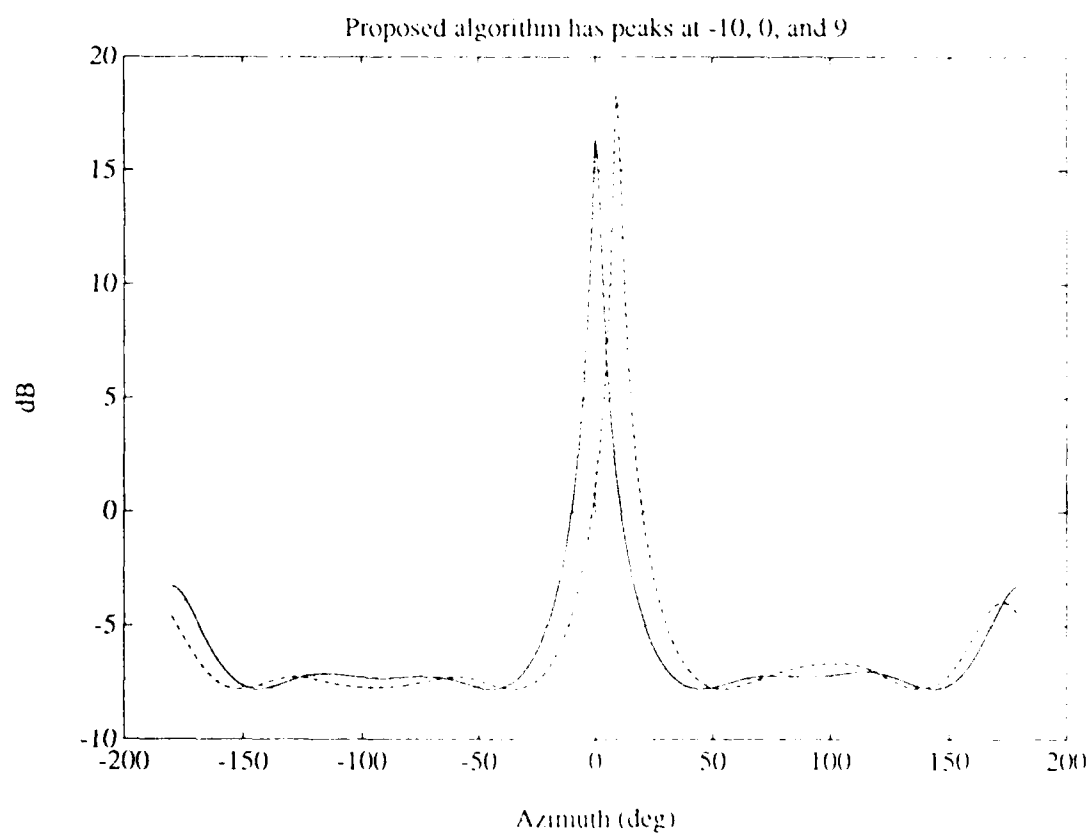


Figure 5: Spectrum of the New Method

## 5 Summary and Observations

This report has presented a two-part method for estimating the directions of arrival of uncorrelated narrowband sources when there are arbitrary phase errors and angle-independent gain errors. The signal steering vectors are estimated in the first part of the method; in the second part, the arrival directions are estimated. It should be noted that the second part of the method can be tailored to incorporate additional information about the nature of the phase errors. For example, if the phase errors are known to be caused solely by element misplacement, the element locations can be estimated concurrently with the DOAs by trying to match the theoretical steering vectors to the estimated ones. Simulation results suggest that, for general perturbation, the method can resolve closely spaced sources under conditions for which a standard high-resolution DOA method such as MUSIC fails.

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